Supersymmetry breaking by fermions and its restoration at high temperature

1993 J. Phys. A: Math. Gen. 26 4463

(http://iopscience.iop.org/0305-4470/26/17/050)

J. Phys. A: Math. Gen. 26 (1993) 4463–4474. Printed in the UK

Supersymmetry breaking by fermions and its restoration at high temperature

He-Shan Song†‖, Guang-Nan Xu‡ and Ing An§

† Center of Theoretical Physics, CCAST (World Laboratory), PO Box 8730, Beijing 100080, People's Republic of China

‡ Department of Physics, Liaoning Normal University, Dalian 116021, People's Republic of China

§ Institute of Theoretical Physics, Academia Sinica, Beijing 100080, People's Republic of China

Received 23 July 1992, in final form 11 March 1993

Abstract. It is shown, in terms of the Nambu–Jona–Lasinio theory and the Coleman–Weinberg mechanism, that supersymmetry breaking can be realized dynamically due to radiative correction even in the Wess–Zumino model. Supersymmetry behaviour at finite temperatures is also investigated and it is shown that the supersymmetry broken dynamically at zero temperature can be restored at finite temperatures.

1. Introduction

The spontaneous breaking of supersymmetry and its behaviour at finite temperatures has been investigated by many authors [1–9]. The usual method for realizing spontaneous breaking of supersymmetry is adding a gauge-invariant but parity-violating term [2] ξD (Feyet–Iliopoulos term) to the Lagrangian. The adding of the Feyet–Iliopoulos term to the Lagrangian leads to mass splitting between bosons and fermions in the supermultiplets and, hence, the supersymmetry breaks down spontaneously. It has been shown, however, that spontaneous breaking of supersymmetry does not occur for the interaction of a chiral superfield with itself [2] (such as the Wess–Zumino model). Spontaneous breaking of supersymmetry may occur only for the interaction of more than one, say N, chiral superfields [4] in the absence of any gauge fields.

A parallel development is the study of supersymmetry behaviour at finite temperatures. Some authors have shown [5, 6] that at zero temperature the supersymmetry is not easy to break spontaneously, but that finite temperatures automatically break the supersymmetry. They argued that the supersymmetry broken at zero temperature cannot be restored at finite temperatures. Finite temperatures always break supersymmetry.

However, since supersymmetry is so special, one would like to somehow maintain it at high temperatures [10]. This prompted Van Hove to propose [7] a modified definition of order parameters at finite temperatures and to examine supersymmetry

‖ Address for correspondence: Department of Physics, Dalian University of Technology, Dalian 116024, People's Republic of China.

behaviour at finite temperatures. Unfortunately, this modified definition of order parameters still does lead to the expected behaviour of supersymmetry at finite temperatures.

Recently, however, some authors [11–15] have again studied supersymmetry breaking and its behaviour at finite temperatures. Cahill [11] has shown, in terms of the Coleman–Weinberg (CW) mechanism) [16], which is based on the method of effective potential and putting in a three-dimensional momentum cut-off, that the fermion zero-point energy (radiative corrections) in the effective potential breaks the supersymmetry in the massless Wess–Zumino model.

The purpose of the present paper is to investigate the behaviour of supersymmetry at finite temperatures in terms of the Nambu–Jona-Lasinio (NJL) theory [17] and the CW mechanism. The key point of our method is to establish the self-consistency equation for the order parameter rather than to define the effective potential at finite temperatures and to solve the self-consistency equation by putting in the momentum cut-off. We will show that the supersymmetry broken at zero temperature can be restored at a finite temperature.

The paper is organized as follows. In section 2 we will briefly review the NJL theory and show, in terms of the theory and the CW mechanism, that the contribution of the self-energy part of the fermion to the self-consistency equation for the order parameter will generate different dynamical masses for different fields and leads to mass splitting between fermions and bosons. The non-vanishing vacuum expectation of the order parameter also leads to the non-vanishing vacuum expectation value of an auxiliary field, and the supersymmetry should be broken down.

In section 3 we will investigate supersymmetry behaviour at finite temperatures and shown, by solving the self-consistency equation for the order parameter at finite temperatures, that the supersymmetry which is broken dynamically at zero temperature can be restored at a critical temperature $T_c=\sqrt{3}\Lambda/\pi$, where Λ denotes the momentum cut-off. Next, we will study the critical temperature for the massive Wess–Zumino model by dimensional analysis and find $T_c\sim[\Lambda^2/\pi^2+am_0^2/g^2]^{1/2}$, with m_0 being the original mass of the fields in the model. Finally, we summarize our conclusions.

2. NJL theory and supersymmetry breaking

As is well known, Nambu and Jona-Lasinio suggested, in 1961, that the nucleon mass arises largely as a self-energy of some primary massless fermions. The NJL model is based on the Lagrangian density

$$\mathscr{L}=\mathscr{L}_0+\mathscr{L}_1 \tag{1}$$

where $\mathscr{L}_0$ is the free Lagrangian

$$\mathscr{L}_0=\bar{\psi}\mathrm{i}\partial\!\!\!/\psi \tag{2}$$

and $\mathscr{L}_1$ is a four-fermion interaction of the type

$$\mathscr{L}_1=\tfrac{1}{2}g[(\bar{\psi}\psi)^2-(\bar{\psi}\gamma_5\psi)^2]. \tag{3}$$

Instead of diagonalizing $\mathscr{L}_0$ and treating $\mathscr{L}_1$ as a perturbation, Nambu and Jona-Lasinio introduced a self-energy Lagrangian $\mathscr{L}_s=\delta m\bar{\psi}\psi$ and rewrote the Lagrangian (1) as

$$\mathscr{L}=(\mathscr{L}_0+\mathscr{L}_s)+(\mathscr{L}_1-\mathscr{L}_s). \tag{4}$$

The crucial assumption of the NJL model is that, despite vanishing of the bare fermion mass, the physical mass m of the fermion is non-zero. Then, by using Dyson's mass renormalization prescription, the Lagrangian can be written as

$$\mathscr{L}=\mathscr{L}_0'+\mathscr{L}_1' \tag{5}$$

with

$$\mathscr{L}_0'=\bar{\psi}(\mathrm{i}\partial\!\!\!/-m)\psi \tag{6}$$

$$\mathscr{L}_1'=\mathscr{L}_1+\delta m\bar{\psi}\psi. \tag{7}$$

The self-mass δm is given by

$$\delta m=\sum{}^{*}(P)|_{P=m} \tag{8}$$

and the right-hand side of equation (8) can be calculated, to the first order in g, from the self-energy diagram of the fermion. Since $m_0=0$, we have $m=\delta m$ and so one obtains the self-consistent equation for the physical mass of the fermion:

$$m=\sum{}^{*}(P)|_{P\!\!\!/=m}. \tag{9}$$

Nambu and Jona-Lasinio evaluated

$$m\approx 2\mathrm{i}g\,\mathrm{Tr}\,S_\mathrm{F}(0)=8gm\int\frac{\mathrm{d}^4P}{(2\pi)^4}\frac{\mathrm{i}}{P^2-m^2} \tag{10}$$

or, assuming $m\neq 0$,

$$1=8g\int\frac{\mathrm{d}^4P}{(2\pi)^4}\frac{\mathrm{i}}{P^2-m^2}. \tag{11}$$

In equation (10)

$$S_\mathrm{F}(0)=\int\frac{\mathrm{d}^4P}{(2\pi)^4}\frac{1}{P^2-m^2} \tag{12}$$

represents the contribution of the closed-loop (self-energy) diagram of fermions.

Equation (11) gives

$$\frac{2\pi^2}{g\Lambda^2}=1-\frac{m^2}{\Lambda^2}\ln\left(\frac{\Lambda^2}{m^2}+1\right) \tag{13}$$

by introducing the invariant momentum cut-off $P^2=\Lambda^2$ [16, 17]. Since the right-hand side of equation (13) is positive and $\leqslant 1$ for real Λ and m, one gets the constraint inequality

$$0<2\pi^2/g\Lambda^2<1. \tag{14}$$

We see that in the NJL theory, starting from a massless fermion, one generates the physical mass of a fermion self-consistently.

Lurie and Macfarlanf [18] have shown the equivalence between Lagrangian field theory of the four-fermion type considered by Nambu and Jona-Lasinio and a Lagrangian theory of the same fermion fields with coupling of the Yukawa type,

$$\mathscr{L}_\mathrm{Y}=\mathscr{L}_0+G\bar{\psi}\psi\Phi_\mathrm{s}+G\bar{\psi}\gamma_5\psi\Phi_\mathrm{P} \tag{15}$$

and obtained the fermion mass in the equivalent Yukawa theory in the same way.

Consider now the massless Wess–Zumino model of supersymmetry described by the Lagrangian density [19]

$$\mathscr{L}=\mathscr{L}_0+\mathscr{L}_g \tag{16}$$

with

$$\mathscr{L}_0=\tfrac{1}{2}[(\partial_\mu A)^2+(\partial_\mu B)^2+\mathrm{i}\bar{\psi}\not{\partial}\psi+F^2+G^2] \tag{17}$$

$$\mathscr{L}_g=-g[(A^2-B^2)F+2ABG+\bar{\psi}(A-\gamma_5 B)\psi]. \tag{18}$$

In equations (17) and (18), A and B are, respectively, a scalar and a pseudoscaler field, ψ is a Majorana spinor and F, G are auxiliary fields.

The field equations for the auxiliary fields are given by

$$F=g(A^2-B^2) \tag{19}$$

$$G=2gAB \tag{20}$$

which can be used to eliminate the auxiliary fields from the Lagrangian. The result of their elimination is

$$\mathscr{L}=\tfrac{1}{2}[(\partial_\mu A)^2+(\partial_\mu B)^2+\mathrm{i}\bar{\psi}\not{\partial}\psi]-\tfrac{1}{2}g^2(A^2+B^2)^2-g\bar{\psi}(A-\gamma_5 B)\psi. \tag{21}$$

We now introduce the external sources $J_A(x)$, $J_B(x)$, $\bar{J}_\psi(x)$ and $J_\psi(x)$ coupled to the fields A, B, ψ and $\bar{\psi}$, respectively. Then the Lagrangian becomes

$$\mathscr{L}[J]=\mathscr{L}+J_A A+J_B B+\bar{J}_\psi\psi+\bar{\psi}J_\psi \tag{22}$$

and the equations of motion following from the Lagrangian (22) are given by

$$\Box A+2g^2A(A^2+B^2)=-g\bar{\psi}\psi+J_A(x) \tag{23}$$

$$\Box \mathrm{B}+2g^2B(A^2+B^2)=g\bar{\psi}\gamma_5\psi+J_B(x) \tag{24}$$

$$[\mathrm{i}\partial-2g(A-\gamma_5 B)]\psi=-2J_\psi(x). \tag{25}$$

As usual, the generating functional is given by

$$Z[J]=\int[\mathrm{d}A][\mathrm{d}B][\mathrm{d}\psi][\mathrm{d}\bar{\psi}]\exp\left(\frac{\mathrm{i}}{\hbar}\int \mathrm{d}^4x\mathscr{L}[J]\right). \tag{26}$$

By using the generating functional $Z[J]$. the vacuum expectation value of a field, say $A(x)$, in the presence of external sources can be found as

$$\langle A(x)\rangle_0^J=\frac{\delta W[J]}{\delta J_A(x)}=\frac{1}{Z[J]}\int[\mathrm{d}A][\mathrm{d}B][\mathrm{d}\psi][\mathrm{d}\bar{\psi}]A(x)\exp\left(\frac{\mathrm{i}}{\hbar}\int \mathrm{d}^4x\mathscr{L}[J]\right) \tag{27}$$

with

$$W[J]=(\hbar/\mathrm{i})\ln Z[J]. \tag{28}$$

Let us now take vacuum expectation values of equations (23) and (24). Then one gets

$$\Box\langle A\rangle_0^J+2g^2\langle A^3\rangle_0^J+2g^2\langle AB^2\rangle_0^J=-g\langle\bar{\psi}\psi\rangle_0^J+J_A(x) \tag{29}$$

$$\Box\langle B\rangle_0^J+2g^2\langle BA^2\rangle_0^J+2g^2\langle B^3\rangle_0^J=g\langle\bar{\psi}\gamma_5\psi\rangle_0^J+J_B(x). \tag{30}$$

By using equation (27), it is easy to calculate the vacuum expectation values of the terms $\langle A^3\rangle_0^J$, $\langle AB^2\rangle_0^J$, $\langle BA^2\rangle_0^J$ and $\langle B^3\rangle_0^J$, and to the lowest-order approximation in $\hbar$, equations (29) and (30) become

$$2g^2A_0^3+2gA_0B_0^3=-\langle\bar{\psi}\psi\rangle=\mathrm{iTr}S_{\mathrm{F}}(0) \tag{31}$$

$$2gB_0^3+2gB_0A_0^3=\langle\bar{\psi}\gamma_5\psi\rangle=-\mathrm{iTr}\gamma_5S_{\mathrm{F}}(0)] \tag{32}$$

when we take the limit $J\to 0$. In equations (31) and (32),

$$A_0=\langle A\rangle_0^J|_{J=0} \tag{33}$$

$$B_0=\langle B\rangle_0^J|_{J=0} \tag{34}$$

and

$$\mathrm{Tr}S_{\mathrm{F}}(0)=\mathrm{i}\langle\bar{\psi}(x)\psi(x)\rangle_0^J|_{J=0} \tag{35}$$

$$\mathrm{Tr}[\gamma_5S_{\mathrm{F}}(0)]=\mathrm{i}\langle\bar{\psi}(x)\gamma_5\psi(x)\rangle_0^J|_{J=0}. \tag{36}$$

The right-hand sides of equations (31) and (32) represent closed-loop diagrams which correspond to the self-energy of the fermion. The self-energy of the fermion can be found from equation (25). From equation (25) we obtain Schwinger's functional differential equation

$$\begin{aligned}\mathrm{i}\not{\partial}S_{\mathrm{F}}^J(x,y)&=\delta^4(x-y)+2g\left(\langle A\rangle_0^JS_{\mathrm{F}}^J(x,y)+\frac{\hbar}{\mathrm{i}}\frac{\delta S_{\mathrm{F}}^J(x,y)}{\delta J_A(x)}\right)\\&\quad-2g\left(\langle B\rangle_0^J\gamma_5S_{\mathrm{F}}^J(x,y)+\frac{\hbar}{\mathrm{i}}\gamma_5\frac{\delta S_{\mathrm{F}}^J(x,y)}{\delta J_B(x)}\right)+2\mathrm{i}J_\psi(x)\langle\bar{\psi}(y)\rangle_0^J.\end{aligned} \tag{37}$$

To the lowest order in $\hbar$ and in the limit $J\to 0$, equation (37) reduces to

$$[\mathrm{i}\not{\partial}-2g(A_0-\gamma_5B_0)]S_{\mathrm{F}}(x-y)=\delta^4(x-y). \tag{38}$$

So the fermion propagator in the momentum space can be written as

$$S_{\mathrm{F}}(P)=\frac{1}{\mathbf{P}-2g(A_0-\gamma_5B_0)}. \tag{39}$$

Substituting equation (39) into equations (31) and (32) yields

$$2gA_0(A_0^2+B_0^2)=\mathrm{iTr}\int\frac{\mathrm{d}^4P}{(2\pi)^4}\frac{\mathrm{i}}{\mathbf{P}-2g(A_0-\gamma_5B_0)} \tag{40}$$

$$2gB_0(A_0^2+B_0^2)=-\mathrm{iTr}\int\frac{\mathrm{d}^4P}{(2\pi)^4}\frac{\mathrm{i}\gamma_5}{\mathbf{P}-2g(A_0-\gamma_5B_0)}. \tag{41}$$

The straightforward calculation shows that equations (40) and (41) can be simplified to the same equation:

$$A_0^2+B_0^2=4\int\frac{\mathrm{d}^4P}{(2\pi)^4}\frac{\mathrm{i}}{P^2-4g^2(A_0^2+B_0^2)}. \tag{42}$$

We see from equation (42) that it determines only the magnitude of $\boldsymbol{C}_0=(A_0,B_0)$, not its direction. The direction is essentially arbitrary. The arbitrariness in the direction of $\boldsymbol{C}_0$ reflects infinite degeneracy of the vacuum state. If we take $\boldsymbol{C}_0$ to lie along a particular direction, say the A_0 direction, then equation (42) becomes simply an equation for A_0 ($B_0=0$):

$$A_0^2=4\int\frac{d^4P}{(2\pi)^4}\frac{i}{P^2-4g^2A_0^2}. \tag{43}$$

The frame in which $B_0=0$ is characterized by a vacuum state which is an eigenstate of parity. For the space inversion invariance to hold (note that our original Lagrangian (21) is invariant under the space inversion transformation), here and afterwards we take $B_0=0$. Equation (43) is a self-consistency equation for the order parameter A_0. If we set

$$m_\psi^2=4g^2A_0^2 \tag{44}$$

in equation (43), it reduces to the self-consistency equation for the physical fermion mass m_ψ:

$$m_\psi^2=16g^2\int\frac{d^4P}{(2\pi)^4}\frac{i}{P^2-m_\psi^2}. \tag{45}$$

We see from equation (45) that starting from a massless fermion one generates the physical mass self-consistently via the CW mechanism.

The integration in equation (45) is quadratically divergent. Introducing a momentum cut-off Λ and performing Wick rotation one can make the integration meaningful. The result is given by

$$m_\psi^2=\frac{g^2}{\pi^2}\left(\Lambda^2-m_\psi^2\ln\frac{\Lambda^2}{m_\psi^2}\right) \tag{46}$$

or

$$A_0^2=\frac{\Lambda^2}{4\pi^2}\left(1-\frac{4g^2A_0^2}{\Lambda^2}\ln\frac{\Lambda^2}{4g^2A_0^2}\right). \tag{47}$$

Translating the scalar field A as

$$A\to A'=A-A_0 \tag{48}$$

and rewriting the Lagrangian (21) in terms of $A+A_0$ instead of A, one obtains

$$\begin{aligned}\mathcal{L}=\tfrac{1}{2}[(\partial_\mu A)^2+(\partial_\mu B)^2+i\bar\psi\not\partial\psi-m_A^2A^2-m_B^2B^2-m_\psi\bar\psi\psi]-\tfrac{1}{2}g^2(A^2+B^2)^2\\ -g\bar\psi(A-\gamma_5B)\psi-2g^2A_0A(A_0^2+A^2+B^2)\end{aligned} \tag{49}$$

with

$$\begin{aligned}m_A^2&=6g^2A_0^2\\ m_B^2&=2g^2A_0^2\\ m_\psi&=2gA_0.\end{aligned} \tag{50}$$

Evidently, the boson fields A, B and the fermion field ψ have acquired different masses m_A, m_B and m_ψ, respectively, due to the non-vanishing vacuum expectation value of the field A which is determined by the self-energy of the fermion.

We see from equation (50) that the masses m_A, m_B and m_ψ satisfy the Ferrara–Giradello–Palumbo mass formula [20]

$$\sum_J (-1)^{2J}(2J+1)m_J^2 = 6g^2A_0^2 + 2g^2A_0^2 - 8g^2A_0^2 = 0. \tag{51}$$

Here m_J^2 is the squared masses associated with a field of spin J.

It is worth noting that, apart from the solution $A_0^2+B_0^2 \neq 0$, equations (40) and (41) also admit the trivial solution $A_0 = B_0 = 0$ which preserves supersymmetry. A natural question that then arises is whether the non-trivial solution $A_0^2+B_0^2 \neq 0$ corresponds to the minimum of the effective potential. This correspondence can be verified by using the effective potential [16]. For the massless Wess–Zumino model the effective potential is given by

$$V_{\text{eff}} = \tfrac{1}{2}g^2(A^2+B^2)^2 + \tfrac{1}{2}\sum_J (-1)^{2J}(2J+1)\int \frac{\mathrm{d}^3P}{(2\pi)^3}(P^2+m_J^2)^{1/2}. \tag{52}$$

The momentum integral in equation (52) can be found by introducing the momentum cut-off $|P| = \Lambda$. Cahill has shown [11], through numerical calculation, that the effective potential (52) has a minimum at $A_0^2+B_0^2 \neq 0$. So we conclude that in the massless Wess–Zumino model the self-energy of the fermion gives masses to the particles and moves the minimum of the effective potential away from zero so as to break the supersymmetry.

From equations (19) and (20) we find the expectation value of the auxiliary fields:

$$\langle F\rangle = g\langle A^2\rangle \approx gA_0^2 \neq 0 \tag{53}$$

$$\langle G\rangle = 2g\langle A\rangle\langle B\rangle = 0. \tag{54}$$

The only case where $\langle F\rangle = \langle G\rangle = 0$ is $A_0 = B_0 = 0$. The non-vanishing expectation value of the auxiliary field F is further evidence of supersymmetry breaking.

Note that, in our model, there is no Goldstone boson or fermion after the supersymmetry breakdown. This result is very similar to symmetry breaking in the linear σ-model. In the linear σ-model the linear term $C\sigma$ induces symmetry breaking (externally) but there is no Goldstone boson.

It has also been claimed [6] that there are no Goldstone fermions associated with supersymmetry breaking.

3. Supersymmetry restoration at finite temperatures

In the previous section we have established the self-consistency equation for the order parameter A_0 and examined dynamical breaking of supersymmetry. If a corresponding equation at finite temperatures can be established then we can investigate supersymmetry behaviour at finite temperatures by solving the self-consistency equation. We wish to find such a temperature T_c at which the order parameter $A_0(\beta)$

reduces to zero and the mass splitting between bosons and fermions will be eliminated.

It is well known that, at finite temperatures, all physically interesting quantities such as Green's functions in a system are given not by the vacuum-to-vacuum transition amplitude as in the usual field theories, but by the statistical average defined by [21]

$$\langle \ldots \rangle = \frac{\mathrm{Tr}[\exp(-\beta H) \ldots]}{\mathrm{Tr}\exp(-\beta H)} \tag{55}$$

where β is proportional to the inverse of temperature and H denotes the Hamiltonian of the system. By using the field theory at finite temperature [22], the statistical average of the scalar field A and the fermion propagator $S_F^J(x-y)$ in the presence of an external source can be written, respectively, as

$$A_0^J(\beta) = \frac{1}{Z^\beta[J]} \int [\mathrm{d}A][\mathrm{d}B][\mathrm{d}\psi][\mathrm{d}\bar{\psi}]A(x)\exp\left(\int_0^\beta \mathrm{d}\tau \int \mathrm{d}^3x \mathscr{L}_E[J]\right) \tag{56}$$

$$S_F^J(x-y,\beta) = \frac{1}{Z^\beta[J]} \int [\mathrm{d}A][\mathrm{d}B][\mathrm{d}\psi][\mathrm{d}\bar{\psi}]\psi(x)\bar{\psi}(x)\exp\left(\int_0^\beta \mathrm{d}\tau \int \mathrm{d}^3x \mathscr{L}_E[J]\right) \tag{57}$$

with

$$Z^\beta[J] = \int [\mathrm{d}A][\mathrm{d}b][\mathrm{d}\psi][\mathrm{d}\bar{\psi}]\exp\left(\int_0^\beta \mathrm{d}\tau \int \mathrm{d}^3x \mathscr{L}_E[J]\right). \tag{58}$$

Here $\mathscr{L}_E[J]$ denotes the Lagrangian $\mathscr{L}[J]$ in Euclidean space. Here and afterwards we take $\hbar = 1$.

The important observation in field theory at finite temperatures is the fact that the finite-temperature Green's functions satisfy the same differential equations as the zero-temperature Green's functions except that they satisfy periodic (anti-periodic for the fermion case) boundary condition for an imaginary time τ, and the momentum $\boldsymbol{P} = (P_0, \boldsymbol{P})$ has to be replaced by

$$P_n = (\omega_n, \boldsymbol{P}) \tag{59}$$

with

$$\omega_n = (2n+1)\pi/\beta \qquad (n \text{ integer}) \tag{60}$$

for the fermions.

So, at finite temperatures, the self-consistency equation (43) changes to [23, 24]

$$A_0^2(\beta)=\frac{4}{\beta}\sum_{n=-\infty}^{\infty}\int\frac{d^3P}{(2\pi)^3}\frac{1}{\omega_n^2+P^2+4g^2A_0^2(\beta)}=\frac{1}{2\pi^3\beta}\sum_{n=-\infty}^{\infty}\int\frac{d^3P}{\omega_n^2+E^2} \tag{61}$$

with

$$E^2=P^2+4g^2A_0^2(\beta). \tag{62}$$

We first calculate the summation in equation (61):

$$\sum_{n=-\infty}^{\infty}\frac{1}{\omega_n^2+E^2}=\sum_{n=-\infty}^{\infty}\frac{1}{(2n+1)^2\pi^2/\beta^2+E^2}=\frac{\beta^2}{\pi^2}\sum_{n=-\infty}^{\infty}\frac{1}{(2n+1)^2+a^2} \tag{63}$$

with $a^2=(\beta E/\pi)^2$. The summation in equation (63) can be simplified further to

$$\sum_{n=-\infty}^{\infty}\frac{1}{(2n+1)^2+a^2}=2\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2+a^2}=2\sum_{n=1}^{\infty}\frac{1}{n^2+a^2}-\frac{1}{2}\sum_{n=1}^{\infty}\frac{1}{n^2+(a/2)^2}. \tag{64}$$

By using

$$\sum_{n=1}^{\infty}\frac{\cos nx}{n^2+a^2}=\frac{\pi}{2a}\frac{\cosh a(\pi-x)}{\sin ha\pi}-\frac{1}{2a} \tag{65}$$

and

$$\coth 2x=(\tanh x+\coth x)/2 \tag{66}$$

one gets

$$\sum_{n=-\infty}^{\infty}\frac{1}{(2n+1)^2+a^2}=\frac{\pi}{2a}\tanh\frac{a\pi}{2}=\frac{\pi}{2a}\left(1-\frac{2}{\exp(a\pi)+1}\right)=\frac{\pi^2}{2\beta}\left(\frac{1}{E}-\frac{2}{E[\exp(\beta E)+1]}\right). \tag{67}$$

Thus, the self-consistency equation at finite temperature, equation (61), can be reduced to

$$A_0^2(\beta)=\frac{1}{4\pi^3}\int d^3p\left(\frac{1}{E}-\frac{2}{E[\exp(\beta E)+1]}\right)=\frac{1}{4\pi^3}(\mathrm{I}-\mathrm{II}) \tag{68}$$

with

$$\mathrm{I}=\int\frac{d^3p}{E}=4\pi\int_0^{\infty}\frac{|p|^2d|p|}{(p^2+4g^2A_0^2(\beta))^{1/2}} \tag{69}$$

$$\mathrm{II}=2\int\frac{d^3p}{E[\exp(\beta E)+1]}=8\pi\int_0^{\infty}\frac{|p|^2d|p|}{(p^2+4g^2A_0^2(\beta))^{1/2}\{\exp[\beta(p^2+4g^2A_0^2(\beta))^{1/2}]+1\}}. \tag{70}$$

The integration in equation (69) is divergent. As before, introducing a momentum cut-off Λ one can make the integration finite. The result is

$$\mathrm{I}=2\pi\left(\Lambda(\Lambda^2+4g^2A_0^2(\beta))^{1/2}+4g^2A_0^2(\beta)\ln\left|\frac{2gA_0(\beta)}{\Lambda+(\Lambda^2+4g^2A_0^2(\beta))^{1/2}}\right|\right). \tag{71}$$

In order to calculate the integration (70), set $x=|p|\beta$, then the integration (70) can be written as

$$\Pi=\frac{8\pi}{\beta^2}\int_0^\infty \frac{x^2\,\mathrm{d}x}{\sqrt{(x^2+y^2)}\{\exp[\sqrt{(x^2+y^2)}]+1\}} \tag{72}$$

with

$$y=2\beta g A_0(\beta). \tag{73}$$

We are interested in supersymmetry behaviour at high temperatures and wish to find the critical temperature T_c at which the order parameter $A_0(\beta)$ tends to zero. So the integration (72) can be calculated in the approximation $y=0$ [24]. In this approximation the integration (72) turns out to be

$$\Pi=\frac{8\pi}{\beta}\int_0^\infty \frac{x\,\mathrm{d}x}{\mathrm{e}^x+1}=\frac{8\pi}{\beta^2}\int_0^\infty \frac{x\,\mathrm{d}x}{\mathrm{e}^x(1+\mathrm{e}^{-x})}. \tag{74}$$

Using

$$\frac{1}{1+\mathrm{e}^{-x}}=\sum_{n=0}^{\infty}(\mathrm{e}^{-2nx}-\mathrm{e}^{-(2n+1)x}) \tag{75}$$

$$\Pi=\frac{8\pi}{\beta^2}\sum_{n=0}^{\infty}\left(\frac{1}{(2n+1)^2}-\frac{1}{4(n+1)^2}\right)=\frac{2\pi^2}{3\beta^2}. \tag{76}$$

Substituting equations (71) and (76) into equation (68) and taking the limit $A_0(\beta)\to 0$, one gets

$$\frac{\Lambda^2}{2\pi^2}-\frac{1}{6\beta_c^2}=0 \tag{77}$$

or

$$T_c=\frac{\sqrt{3}\Lambda}{\pi}. \tag{78}$$

Thus, we have found the critical temperature T_c at which the order parameter $A_0(\beta)$ tends to zero, and the mass splitting between bosons and fermions should be eliminated.

It is worthwhile pointing out that the critical temperature T_c in equation (78) depends only on the momentum cut-off Λ. This is because in the model considered in this paper all the particles are massless. If we start from the massive Wess–Zumino model, the masses of bosons and fermions after supersymmetry breakdown will be given, respectively, by

$$\begin{aligned} m_A^2&=m_0^2+6gm_0A_0+6g^2A_0^2\\ m_B^2&=m_0^2+2gm_0A_0+2g^2A_0^2\\ m_\psi&=m_0+2gA_0 \end{aligned} \tag{79}$$

and we can estimate

$$T_c\sim\left(\frac{\Lambda^2}{\pi^2}+\alpha\frac{m_0^2}{g^2}\right)^{1/2} \tag{80}$$

by dimensional analysis. (This result is very similar to the critical temperature of chiral symmetry restoration. See [23].) Since the second term in equation (80) is proportional to m_0^2/g^2, the critical tempertaure T_c depends only on the momentum cut-off Λ when the mass m_0 in the original Lagrangian is zero.

The supersymmetry restoration at finite temperature can also be shown from equation (19). After supersymmetry breaking has taken place, equation (19) gives

$$F = g(A+A_0)^2 - B^2 = g(A^2 - B^2 + 2AA_0 + A_0^2). \tag{81}$$

Taking the statistical average for both sides of equation (81), one gets

$$\begin{aligned}\langle\langle F\rangle\rangle &= g(\langle\langle A^2\rangle\rangle - \langle\langle B^2\rangle\rangle + 3A_0^2(\beta))\\ &= \frac{g}{\beta}\sum_n\left(\int\frac{\mathrm{d}^3p}{(2\pi)^3}\frac{1}{p_n^2+m_A^2} - \int\frac{\mathrm{d}^3p}{(2\pi)^3}\frac{1}{p_n^2+m_B^2}\right) + 3gA_0^2(\beta)\end{aligned} \tag{82}$$

where $\langle\langle\ldots\rangle\rangle$ denotes the statistical average and

$$\begin{aligned} p_n^2 &= \omega_n^2 + \boldsymbol{p}^2 \qquad & \omega_n &= 2\pi n/\beta \\ m_A^2 &= 6g^2A_0^2(\beta) \qquad & m_B^2 &= 2g^2A_0^2(\beta). \end{aligned} \tag{83}$$

We see from equation (83) that, when $A_0(\beta)\to 0$, the masses $m_A = m_B = 0$ and hence the two integrations in equation (82) will be cancelled mutually for the same momentum cut-off Λ. So the statistical average of the auxiliary field F tends to zero at the critical temperature. This is another sign of supersymmetry restoration at finite temperatures.

In [2], the vacuum expectation value of the auxiliary field D is given by

$$\langle D\rangle = \tfrac{1}{2}gv^2 - \xi \tag{84}$$

where v is the vacuum expectation value of the scalar field and ξ denotes the coefficient of the Feyet–Iliopoulos term. We see from equation (84) that at finite temperature $\langle D(\beta)\rangle$ does not vanish even when $v(\beta) = 0$, because ξ is temperature-independent. Thus, supersymmetry cannot be restored at finite temperatures.

4. Conclusions

We have shown that supersymmetry breaking can be realized via the NJL theory and the CW mechanism even in the Wess–Zumino model of supersymmetry. A different finite-temperature behaviour appears in the theories in which a symmetry is broken dynamically at zero temperature. We have shown, by solving the self-consistency equation for the order parameter at finite temperatures, that supersymmetry which is broken dynamically at zero temperature can be restored at a critical temperature.

Acknowledgment

This work was supported in part by the National Science Foundation of China.

References

[1] Iliopoulos J and Zuminoi B 1974 *Nucl. Phys.* B **76** 310

[2] Feyet P and Iliopoulos J 1974 *Phys. Lett.* **51B** 461
[3] Salam A and Strasdee J 1974 *Phys. Lett.* **49B** 465
[4] O'raifeartaigh L 1975 *Nucl. Phys.* B **96** 331
[5] Das A and Kaku M 1978 *Phys. Rev.* D **18** 4540
[6] Girardello L, Grisaru M T and Salomonson P 1981 *Nucl. Phys.* B **178** 331
[7] Van Hove L 1982 *Nucl. Phys.* B **207** 15
[8] Dicus D A and Tata X R 1984 *Nucl. Phys.* B **239** 237
[9] Fuchs J 1984 *Nucl. Phys.* B **246** 279
[10] Das A 1990 *Preprint UR-1090*
[11] Cahill K 1991 *Phys. Lett.* **269B** 129
[12] Kumar S 1990 *J. Phys. A: Math. Gen.* **23** L127
[13] Cabo A, Chaichian M and Shabad A E 1992 *Preprint* CERN-TH6589/92
[14] Xu G-N and Song H-S 1992 *J. Phys. A: Math. Gen.* **25** 4941
[15] Song H-S and Xu G-N 1993 *J. Phys. A: Math. Gen.* in press
[16] Coleman S and Weinberg E 1973 *Phys. Rev.* D **7** 1888
[17] Nambu Y and Jona-Lasinio G 1964 *Phys. Rev.* **122** 345
[18] Lurie D and Macfarlanf A J 1964 *Phys. Rev.* **136** B816
[19] Ferrara S and Zumino B 1974 *Nucl. Phys.* B **79** 413
[20] Ferrara S, Girradello L and Palumbo F 1979 *Phys. Rev.* D **20** 403
[21] Dolan L and Jackiw R 1974 *Phys. Rev.* D **9** 3320
[22] Bernard C W 1974 *Phys. Rev.* D **9** 3312
[23] Liu Bao-Hua and Li Jia-Rong 1988 *Phys. Rev.* D **37** 190
[24] Linde A D 1979 *Rep. Prog. Phys.* **42** 389